

Lifetime of the superconductive state in short and long Josephson junctions

Giuseppe Augello^{*}, Davide Valenti[†],
Andrey L. Pankratov[‡], Bernardo Spagnolo[§]

*Dipartimento di Fisica e Tecnologie Relative,
Group of Interdisciplinary Physics[¶],
Università di Palermo and CNISM-INFM
Viale delle Scienze, I-90128 Palermo, Italy
Institute for Physics of Microstructures of RAS,
GSP-105, 603950 Nizhny Novgorod, Russia*

Abstract

We study the transient statistical properties of short and long Josephson junctions under the influence of thermal and correlated fluctuations. In particular, we investigate the lifetime of the superconductive metastable state finding the presence of noise induced phenomena. For short Josephson junctions we investigate the lifetime as a function both of the frequency of the current driving signal and the noise intensity and we find how these noise-induced effects are modified by the presence of a correlated noise source. For long Josephson junctions we integrate numerically the sine-Gordon equation calculating the lifetime as a function of the length of the junction both for inhomogeneous and homogeneous bias current distributions. We obtain a nonmonotonic behavior of the lifetime as a function of the frequency of the current driving signal and the correlation time of the noise. Moreover we find two maxima in the nonmonotonic behaviour of the mean escape time as a function of the correlated noise intensity.

I. INTRODUCTION

In recent years a great attention was paid to the study of Josephson junctions (JJs) because of their use both as superconducting quantum bits [1, 2, 3, 4] and nanoscale superconducting quantum interference devices for detecting weak magnetic flux change [5].

^{*}e-mail: augello@gip.dft.unipa.it

[†]e-mail: valentid@gip.dft.unipa.it

[‡]e-mail: alp@ipm.sci-nnov.ru

[§]e-mail: spagnolo@unipa.it

[¶]Electronic address: <http://gip.dft.unipa.it>

This widely used device is very sensitive to magnetic flux change and it is composed by two coupling high- T_c JJs in a superconducting ring. JJs are good candidates to realize superconducting quantum bits (qubits) for quantum information processing. In particular, they were studied at very low temperature in devices making use of charge [6], flux [7] and phase qubits [8]. In JJs, working both at high and low temperatures, the environment affects strongly the behavior of the system. In high temperature superconductors (HTSs) the presence of low-frequency noise, whose intensity is related to the fluctuations in the bias current, temperature and magnetic field, was experimentally found [9]. Also in the low temperature superconductive devices it is very difficult to avoid the influence of environment that constitutes mainly a decoherence source for the system. In particular, the effects on the coherence time of weak current noise in Josephson vortex qubits (JVQs), composed by an uniform long JJ, were investigated [2, 10].

In this framework the study of transient dynamics of JJs in the presence of noise sources is very interesting for the understanding of the interaction between these systems and environment. In particular, the effects of noise strongly influence the current-voltage characteristic of JJs [11, 12, 13]. The dynamics of a JJ is studied considering a fictitious

Brownian particle moving in a washboard potential [14] and the behavior of the current-voltage characteristic of a JJ is strictly related to the lifetime of the superconductive metastable state of the particle. The decay of the particle from the metastable state, in fact, depends on the fluctuations of the voltage across the junction. Recently noise induced effects were experimentally observed in underdamped Josephson junctions [15, 16], and the switching to resistive state of an annular Josephson junction due to thermal activation was analyzed [17].

In the present work, we study the transient dynamics of short overdamped and long JJs under the influence of fluctuating bias current and oscillating potential. We numerically calculate the lifetime of the superconductive metastable state also called the mean switching time (MST) to the resistive state for short JJ (SJJ) and long JJ (LJJ). We demonstrate the presence of noise induced effects such as resonant activation (RA) [18, 19, 20] and noise enhanced stability (NES) [20, 21, 22]. We analyze both the effects of thermal and correlated noise sources. In SJJ and LJJ we consider white noise, accounting for the thermal fluctuations, and correlated (colored) noise separately. Moreover, in LJJ we present an analysis considering together the effects of white and colored noise. For given values of frequency of the

driving signal and suitable noise intensity, we find maxima of the lifetime of the superconductive state. This is an interesting feature for the study of the coherence time of these devices. Our results hold for low temperature superconducting devices when we consider only the effects of the colored noise, and they can be extended to high temperatures, when both colored and white noise come into play.

II. SHORT OVERDAMPED JUNCTIONS

A. Model

The study of SJJs is performed in the framework of the resistively shunted junction (RSJ) model formalism [23]. To take into account the transient dynamics of the system, a fluctuating current term in the RSJ model equation is considered. We obtain the following Langevin equation [24]

$$\frac{d\phi}{dt} = -\omega_c \frac{dU(\phi)}{d\phi} - \omega_c \zeta(t), \quad (1)$$

where ϕ is the order parameter of the system, that is the phase difference of the wave functions in the ground state between left and right superconductive sides of the junction.

The characteristic frequency of the Josephson junction is $\omega_c = 2eR_N I_c / \hbar$, where e is the electron charge, R_N^{-1} is the normal conductivity, I_c is the critical current and $\hbar = h/2\pi$

with h the Plank constant. In Eq. (1) the time is normalized to the inverse of the characteristic frequency of the junction ω_c . In our analysis $\zeta(t)$ is a colored noise source, generated using an Ornstein-Uhlenbeck (OU) process [25], characterized by a correlation time τ_c . The potential profile $U(\phi)$ of Eq. (1) is given by

$$U(\phi) = 1 - \cos\phi - i(t)\phi, \quad (2)$$

where $i(t) = i_0 + f(t)$, $i_0 = i_b/I_c$ is the constant dimensionless bias current and $f(t) = A \sin\omega t$ is the driving current with dimensionless amplitude $A = i_s/I_c$ and frequency ω (i_b and i_s represent the bias current and the driving current amplitude respectively).

The dynamics of a JJ described by Eq. (1) is equivalent to the dynamics of a particle of coordinate ϕ moving in the washboard potential $U(\phi)$. The motion is that of a Brownian particle because of the presence of the noise term [23]. The OU process of Eq. (1) is represented by the stochastic differential equation [25]

$$d\zeta(t) = -\frac{1}{\tau_c} \zeta(t) dt + \frac{\sqrt{\gamma}}{\tau_c} dW(t) \quad (3)$$

where γ is the noise intensity and $W(t)$ is the Wiener process with the usual statistical properties: $\langle dW(t) \rangle = 0$, and $\langle dW(t) dW(t') \rangle = \delta(t - t') dt$. The correlation

function of the OU process is

$$\langle \zeta(t)\zeta(t') \rangle = \frac{\gamma}{2\tau_c} e^{-\frac{|t-t'|}{\tau_c}}. \quad (4)$$

We investigate the dynamics of the Brownian particle in the presence of a time-dependent nonlinear periodic potential, by solving numerically Eq. (1), for $\omega_c=1$.

To study the lifetime of the superconductive state we take, as initial condition for the particle, the minimum of the potential profile, corresponding to the condition $\phi_0 = \arcsin(i_0)$ and we calculate the time spent by the particle to reach the next maximum. We perform a number of simulations ranging from $N = 5000$ to $N = 10000$ to estimate MST with a good approximation. The procedure is repeated for different values of the system parameters such as the amplitude A and frequency ω of the driving current signal, the correlation time τ_c and the intensity γ of the colored noise.

B. Lifetime of metastable state

Our analysis of the lifetime of the superconducting state concerns the behavior of the curves representing MST and the corresponding standard deviation (SD) vs ω , and MST vs γ , for different values of τ_c . In the following figures we present the curves of MST calculated in the presence of colored noise, including also the curves obtained in the pres-

ence of white noise (see also Ref. [24]).

In Fig. 1 we report the behavior of MST vs ω , for two values of the noise intensity, namely $\gamma = 0.02$ and $\gamma = 0.5$, and different values of τ_c . The non-monotonic behavior of the curves shows that the RA phenomenon, already found in the presence of white noise [20, 24], appears also with colored noise. The values of MST around the minimum are influenced by the variation of τ_c , more strongly for higher values of the noise intensity. Moreover, for higher intensity values (see Fig. 1), we find, in a wide range of frequency ($0.3 < \omega < 0.8$), a non-monotonic behavior of MST as a function of the correlation time. This is shown more clearly in Fig. 2, where we report MST vs τ_c for $\omega = 0.7$. In Fig. 3 we report the curves of MST and SD vs ω . We note a range of frequency ($0.2 < \omega < 0.8$) in which MST and SD present a minimum, which indicates the suppression of timing error effects, already found in the presence of white noise [20]. Moreover when we consider the RA phenomenon in the presence of colored noise, we find a scaling effect depending on the values of the correlation time. In Eq. (4) if the correlation time is greater than the characteristic time scale of the system, $\tau_c \gg |t-t'|$, we obtain $\langle \zeta(t)\zeta(t') \rangle \approx \gamma_{\text{colored}}/2\tau_c$. By comparison with the correlation function of the white noise, $\langle \xi(t)\xi(t') \rangle = \gamma_{\text{white}}\delta(t-t')$, we can define the

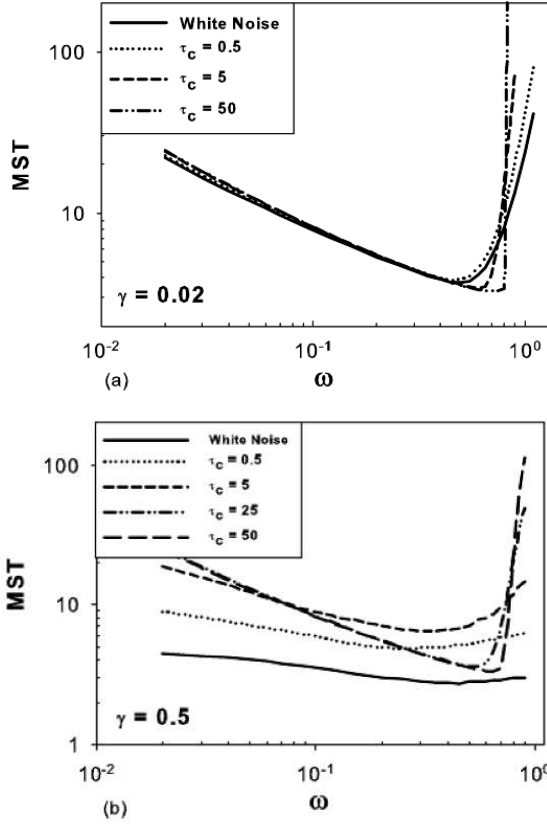


FIG. 1: (a): MST vs ω for white noise and different τ_c , $\gamma=0.02$. (b): MST vs ω for white noise and different τ_c , $\gamma=0.5$. In both panels $i_0=0.8$ and $A=0.7$.

effective intensity of the colored noise scaled by a factor $1/2\tau_c$ and equivalent, in this approximation, to the intensity of the white noise

$$\gamma_{white} = \frac{\gamma_{colored}}{2\tau_c}. \quad (5)$$

We check this equivalence by considering the effect on the system of a white noise with intensity $\gamma_{white}=0.005$ and a colored noise with intensity $\gamma_{colored}=0.5$, setting $\tau_c=50$. In Fig. 4a, we report the curves of MST vs

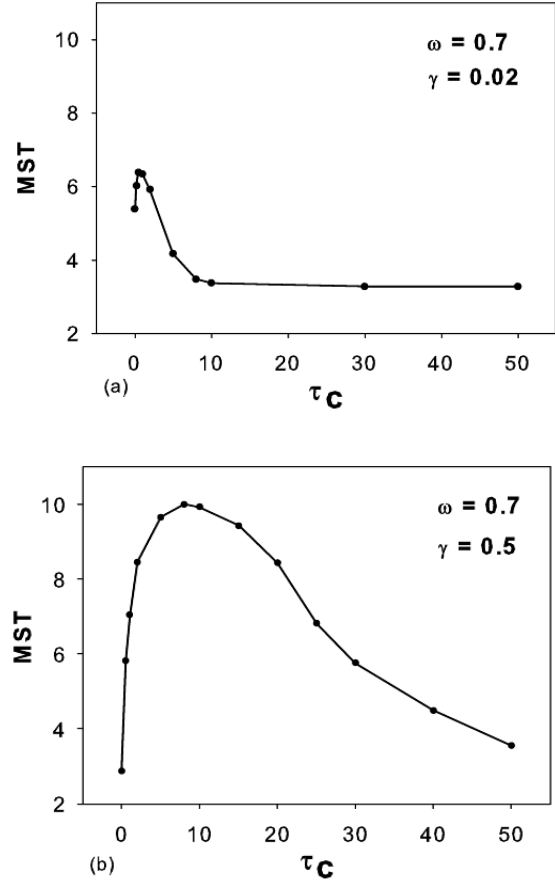


FIG. 2: (a): MST vs τ_c for $\gamma=0.02$. (b): MST vs τ_c for $\gamma=0.5$. In both panels $\omega=0.7$, $i_0=0.8$ and $A=0.7$.

ω calculated for these two values of noise intensity. We see that for MST less than 50 there is a good agreement between the curves. In these conditions the evolution occurs with time scale smaller than the correlation time and the system is not affected by the noise memory. In Fig. 4b, we show the behavior of MST as a function of the noise intensity for different values of the correlation time. We note the appearance of noise

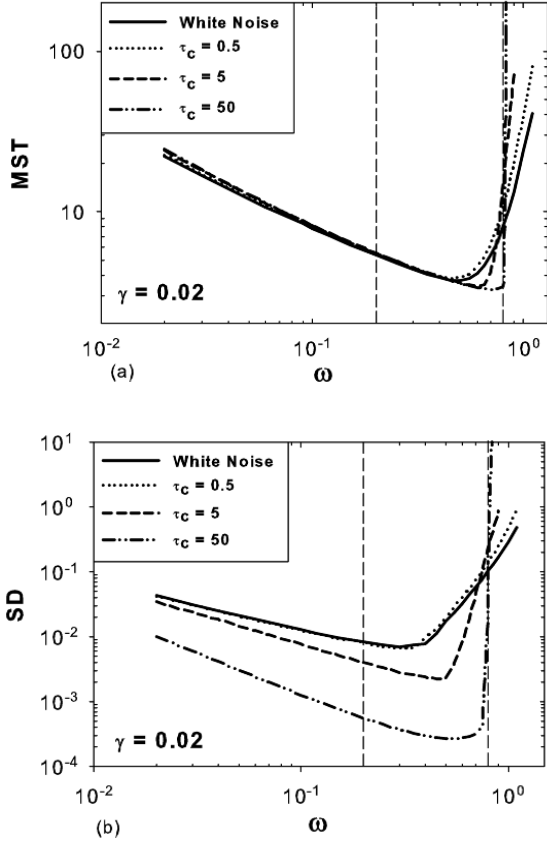


FIG. 3: (a): MST vs ω for different τ_c , $\gamma=0.02$, $i_0=0.8$ and $A=0.7$. (b): Standard deviation (SD) vs ω .

enhanced stability (NES), a phenomenon already found in short JJs in the presence of white noise [20, 24]. Here we observe a range of values of correlation time, namely $0.01 < \tau_c < 1$, in which the curves present a non-monotonic behavior. For $\tau_c=5, 10$, the non-monotonic behavior disappears.

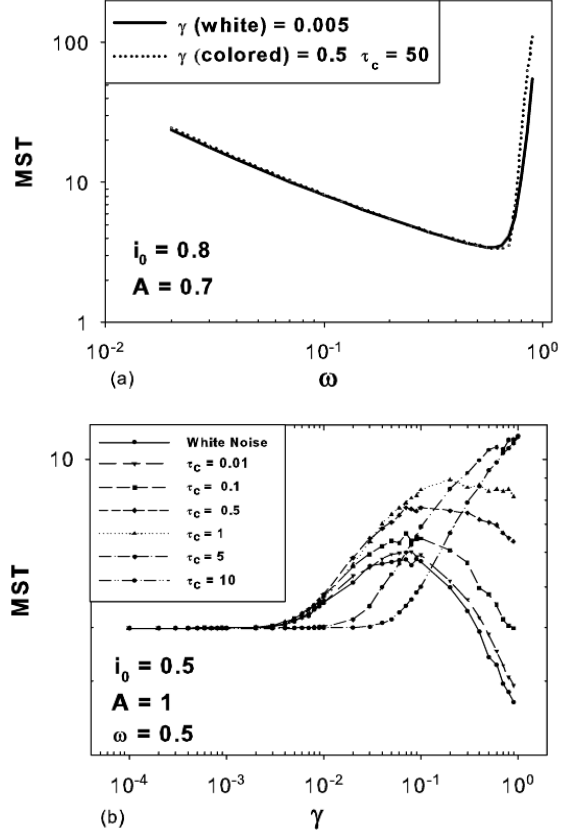


FIG. 4: (a): MST vs ω with $i_0=0.8$, $A=0.7$: white noise (line) and colored noise with $\tau_c=50$ (dots). (b): MST vs γ for white noise and colored noise with different values of τ_c , $\omega=0.5$, $A=1$, $i_0=0.5$.

III. LONG JUNCTIONS

A. Model

The investigation of LJJs is developed in the framework of the sine-Gordon model [23, 26]. The dynamics of a LJJ is described by the motion of a phase string in a one dimensional washboard potential. The transient dynamics is represented by a nonlinear

partial differential equation with a stochastic term [27, 28]

$$\beta \frac{\partial^2 \phi}{\partial t^2} + \frac{\partial \phi}{\partial t} + \frac{\partial^2 \phi}{\partial x^2} = i(x) - \sin \phi + i_f(x, t), \quad (6)$$

where $\beta = \omega_c RC$, with R and C the LJJ equivalent resistance and capacitance, respectively. Here $i(x)$ and $i_f(x, t)$ are the bias current density and the fluctuating current density normalized to the critical current density I_c of the junction, respectively. We treat both the homogeneous bias current density case, namely $i(x) = i_b$, and the inhomogeneous one, namely $i(x) = (i_b L)/(\pi \sqrt{x(L-x)})$, where $L = l/\lambda_J$ with l the length of the junction and λ_J the Josephson penetration depth [27, 28]. In Eq. (6) time is normalized to the inverse of the characteristic frequency, $\omega_c = 2eRI_c/\hbar$, of the junction. Analogously length is normalized to the Josephson penetration depth. The boundary conditions for Eq. (6) are:

$$\frac{\partial \phi(0, t)}{\partial x} = \frac{\partial \phi(L, t)}{\partial x} = 0. \quad (7)$$

As initial position of the phase string in the washboard potential, we consider the minimum, represented by the condition $\phi(x, 0) = \arcsin(i_b)$.

First we present our analysis concerning the effects of a fluctuating current signal described by a correlated noise. Then we consider the effects of both colored and white noise on the system, by inserting in Eq. (6)

a fluctuating term given by the sum of a correlated noisy current signal, $i_{cn}(x, t)$, and a thermal current signal, $i_{wn}(x, t)$, namely

$$i_f(x, t) = i_{cn}(x, t) + i_{wn}(x, t). \quad (8)$$

The integration of Eq. (6) is performed using an implicit finite-difference method [29, 30], that allows to obtain a system of equations, whose solution is calculated by a tridiagonal algorithm. The integration step both for the time and the space is 0.05. The number of simulation ranges from $N = 100$ to $N = 5000$.

B. Effects of correlated noise

Here we study the effects of a correlated noise on the transient dynamics of a LJJ, neglecting, in Eq. (6), the term $i_{wn}(x, t)$ and considering a fluctuating current signal $i_{cn}(x, t)$ with correlation function

$$\langle i_{cn}(x, t) i_{cn}(x', t') \rangle = \frac{(2\gamma_{cn})}{2\tau_c} \delta(x - x') e^{-\frac{|t-t'|}{\tau_c}}, \quad (9)$$

where $2\gamma_{cn}$ is the intensity of the correlated noise. Therefore, the current density $i_{cn}(x, t)$ is subjected to fluctuations that are correlated in time (colored noise), showing a δ -correlated behavior in space (white noise). In Fig. 5 we report the curves of MST vs the length of the junction, in the homogeneous bias current case, for different values of τ_c , including white noise ($\tau_c = 0$) [27, 28]. We

find a decrease of MST as the intensity of the colored noise γ_{cn} increases (see Fig. 5a ($\gamma_{cn} = 0.3$) and Fig. 5b ($\gamma_{cn} = 0.7$)). Moreover, we observe that, for fixed values of the noise intensity, MST increases as the correlation time becomes larger. We also present the numeri-

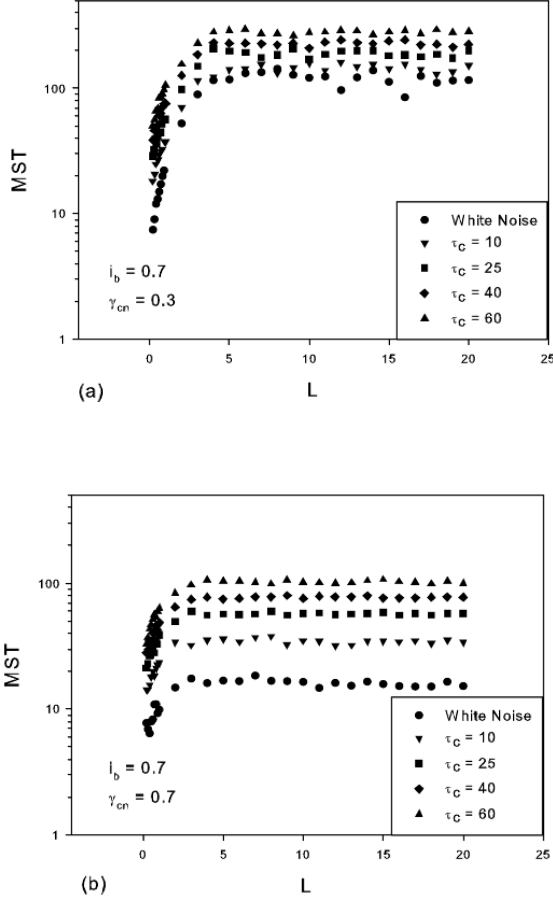


FIG. 5: (a): MST vs L with $i_b=0.7$, $\gamma_{cn}=0.3$. (b): MST vs L with $i_b=0.7$, $\gamma_{cn}=0.7$.

cal results for the case of inhomogeneous bias current. In Fig. 6a we show the behavior of MST as a function of the dimensionless length L . We observe that the curves present a non-monotonic behavior with a maximum

in correspondence of $L = 5$. In order to in-

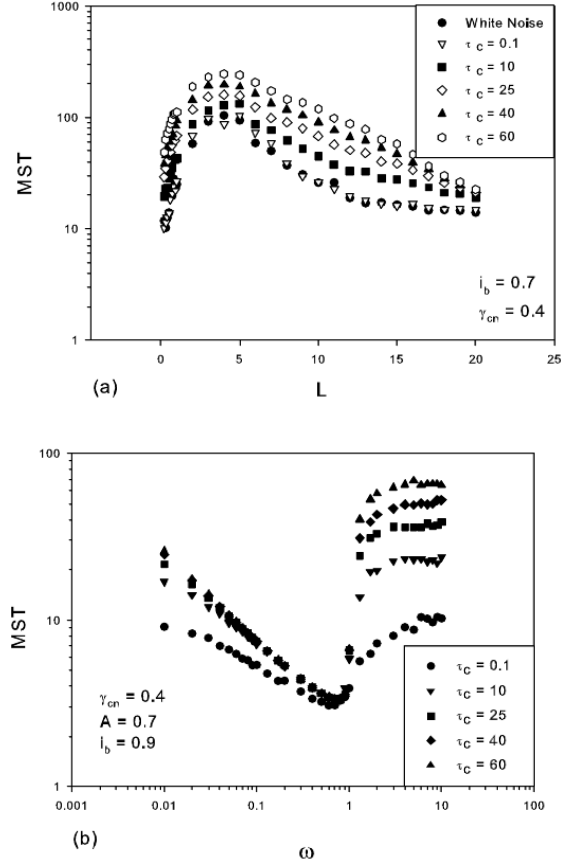


FIG. 6: (a): MST vs L with $i_b=0.7$, $\gamma_{cn}=0.4$. (b): MST vs ω with $\gamma_{cn}=0.4$, $A=0.7$ and $i_b=0.9$,

investigate the presence of RA phenomenon, in Eq. (6) we replace the term $i(x)$ with a homogeneous oscillating driving current signal given by $i(t) = i_b + A \sin(\omega t)$, with A and ω amplitude and angular frequency, respectively, of the periodical driving signal. The curves obtained by numerical simulation are presented in Fig. 6b and Fig. 7a. In both figures we note the presence of a minimum in the curves of MST vs ω . This is the signature of the resonant activation phenomenon.

Moreover in Fig. 7a, in the frequency range

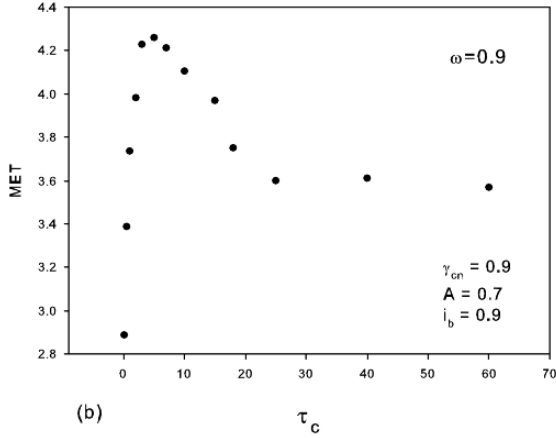
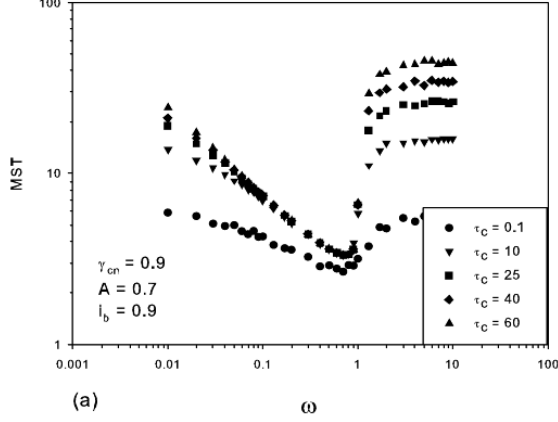


FIG. 7: (a): MST vs ω with $\gamma_{cn}=0.9$, $A=0.7$ and $i_b=0.9$. (b): MST vs τ_c with $\gamma_{cn}=0.9$, $A=0.7$ and $i_b=0.9$.

$0.3 < \omega < 1.0$, we find that curves with different τ_c are overlapping. In order to investigate in detail the behavior in this frequency range we report, for the same parameter values of Fig. 7a, the MST as a function of τ_c for $\omega = 0.9$, finding a nonmonotonic behavior. We also investigate the presence of NES for the homogeneous current case in the resistive state (Fig. 8a) without driving signal,

and in the superconductive state in the presence of a driving current signal (Fig. 8b). The curves of Fig. 8a present a maximum for different values of τ_c . In Fig. 8b we find, in all curves, a double peak. The comparison of the two panels in Fig. 8 shows that the driving signal causes the increase of the two maxima (see Fig. 8b) already present in the absence of periodical term (see Fig. 8a). The non-

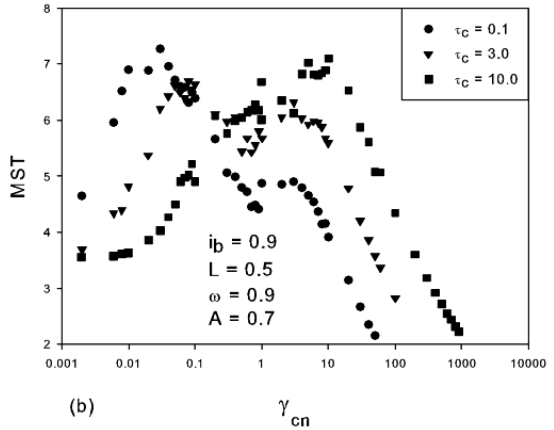
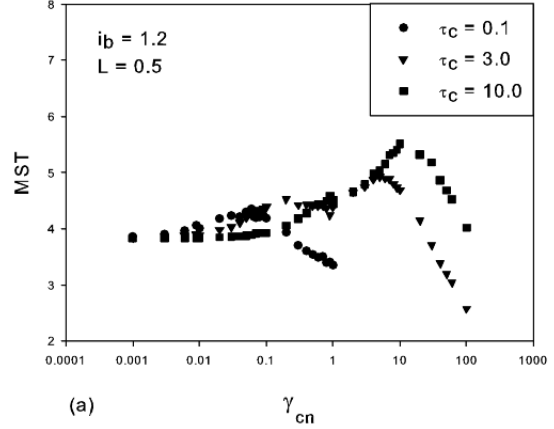


FIG. 8: (a): MST vs γ_{cn} in the resistive state ($i_b = 1.2$) with $L=0.5$ (b): MST vs γ_{cn} in the superconductive state ($i_b = 0.9$) with $L=0.5$, $\omega = 0.9$ and $A=0.7$.

monotonic behavior of MST as a function of the noise intensity γ_{cn} indicates the presence of a NES effect. The curves of Fig. 8 have been calculated for $L = 0.5$. In Fig. 9a, we report the curves of MST vs γ_{cn} for $L = 5$, obtained in the presence of periodical driving signal for the superconductive state. We note that, using the same noise intensities, for longer junction the decay time is shorter. This can be explained recalling that the effect of the spatial correlation disappears on a longer distance. This corresponds to the presence of white noise, which is responsible for faster dynamics and, then, shorter escape time.

C. Effects both of correlated and thermal noise

In this section we investigate the effects both of thermal and correlated noise, considering in Eq. (6) the fluctuating current term $i_f(x, t)$ given by Eq. (8). In Fig. 9b, we present the curves of MST vs γ_{cn} for different white noise intensities γ_{wn} , with $L = 5$.

We note that when γ_{wn} is greater than γ_{cn} the effects of colored noise disappear. If we consider γ_{wn} suitably lower than γ_{cn} , the effects of colored noise become evident. In fact, when γ_{cn} is at least five order of magnitude higher than γ_{wn} , the curve of MST vs γ_{cn} (empty triangles) matches with good

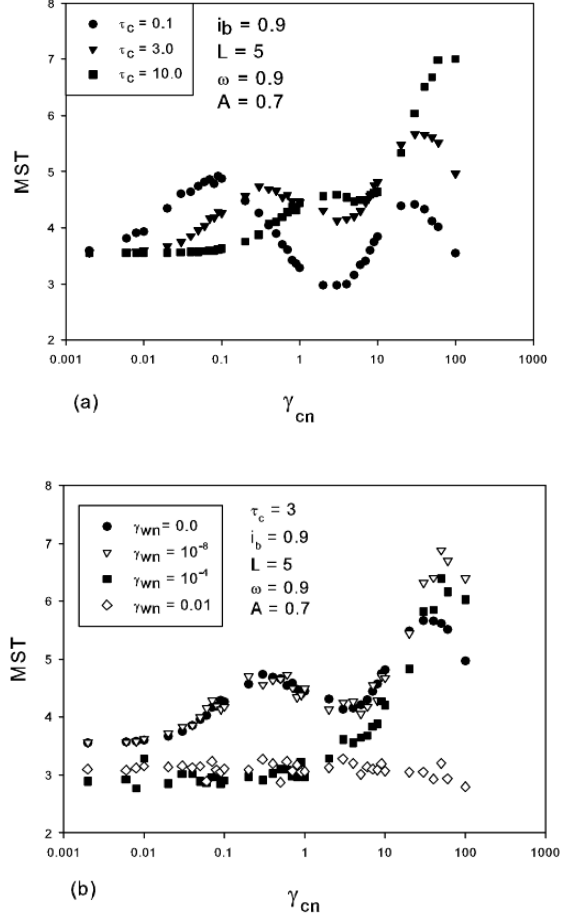


FIG. 9: (a): MST vs γ_{cn} in the superconductive state ($i_b = 0.9$) with $L=5$, for different values of correlation time. (b): MST vs γ_{cn} in the superconductive state ($i_b = 0.9$) with $L=5$, for different values of white noise intensity.

agreement that (black circles) obtained in the absence of white noise (see Fig. 9b for $\gamma_{wn}=10^{-8}$).

IV. CONCLUSIONS

We studied the effects of white and colored noise on the transient dynamics of short and

long Josephson junctions. We investigated the lifetime of the metastable state finding noise induced effects, namely resonant activation and noise enhanced stability. The results obtained throw light upon the role played by different noise sources in the dynamics of superconductive devices, explaining how random fluctuations influence the mean switching time of short and long Josephson junctions. We found that these times are strictly affected by the characteristic parameters of the system such as intensity of noise, frequency of the driving signal and correlation time of the colored noise.

-
- [1] G. Wendin and V. S. Shumeiko, Low Temp. Physics **33** (9), (2007) 724.
 - [2] Ju H. Kim, R. P. Dhungana, KS Park, Phys. Rev. B **73**, (2006) 214506.
 - [3] A. B. Zorin *et al.*, Phys. Rev. B **74**, (2006) 014508.
 - [4] A. J. Berkley *et al.*, Phys. Rev. B **68**, (2003) 060502(R).
 - [5] C. H. Wu *et al.*, Nanotechnology **19**, (2008) 315304.
 - [6] Y. Nakamura *et al.*, Nature, 398, (1999) 786.
 - [7] J. R. Friedman *et al.*, Nature **406**, (2000) 43.
 - [8] J. M. Martinis *et al.*, Phys. Rev. Lett. **89**, (2002) 117901.
 - [9] L. Hao, J.C. Macfarlane, Physica C **292**, (1997) 315.
 - [10] H. Xu *et al.*, Phys. Rev. B **71**, (2005) 064512.
 - [11] A. Marx *et al.*, Phys. Rev. B **51**, (1995) 6735.
 - [12] C. M. Falco *et al.*, Phys. Rev. B **10**, (1974) 1865.
 - [13] J.T. Peltonen, A.V. Timofeev, M. Meschke, and J.P. Pekola, J. Low Temp. Phys. **146**, (2006) 135.
 - [14] Yang Yu *et al.*, Science, **296**, (2002) 889.
 - [15] Y. Yu and S. Han, Phys. Rev. Lett. **91**, (2003) 127003.
 - [16] G. Sun *et al.*, Phys. Rev. E **75**, (2007) 021107.
 - [17] D. Gulevich and F. Kusmartsev, Physica C **435**, (2006) 87.
 - [18] C. R. Doering and J. C. Gadoua, Phys. Rev. Lett. **69**, (1992) 2318; R. N. Mantegna and B. Spagnolo, Phys. Rev. Lett. **84**, (2000) 3025.
 - [19] A. A. Dubkov, N. V. Agudov and B. Spagnolo, Phys. Rev. E **69**, (2004) 061103(7).
 - [20] A. Pankratov and B. Spagnolo, Phys. Rev. Lett. **93**, (2004) 177001(4).
 - [21] R. N. Mantegna and B. Spagnolo, Phys. Rev. Lett. **76**, 563-566 (1996); N. Agudov

- and B. Spagnolo, Phys. Rev. Rap. Comm. E **64**, (2001) 035102(R).
- [22] B. Spagnolo, A. A. Dubkov, N. V. Agudov, Acta Physica Polonica B, Vol. **35** (4), (2004) 1419; A. A. Dubkov, N. V. Agudov and B. Spagnolo, Phys. Rev. E **69**, (2004) 061103(7); A. Fiasconaro, B. Spagnolo, and S. Boccaletti, Phys. Rev. E **72**, (2005) 061110(5); B. Spagnolo *et al.*, Acta Physica Polonica B, Vol. **38** (5), (2007) 1925.
- [23] A. Barone and G. Paternò, *Physics and Application of the Josephson Effect* (Wiley, New York 1982) 122, 146, 264.
- [24] Anna V. Gordeeva *et al.*, Int. J. of Bif. and Chaos, Vol. **18** (9), (2008) 2823.
- [25] C. W. Gardiner, *Handbook of Stochastic Methods*, (Springler-Verlag, Berlin 2004) 106.
- [26] D. R. Gulevich *et al.*, Phys. Rev. Lett. **101**, (2008) 127002; C. Cattuto and F. Marchesoni, Phys. Rev. Lett. **79**, (1997) 5070.
- [27] K. G. Fedorov and A. Pankratov, Phys. Rev. B **76**, (2007) 024504.
- [28] K. G. Fedorov, A. L. Pankratov and B. Spagnolo, Intern. Journ. of Bifurcation and Chaos, Vol. **18**(9), (2008) 2855.
- [29] P. S. Lomdahl, O. H. Soerensen, and P. L. Christiansen, Phys. Rev. B **25**, (1982) 5737.
- [30] Y. Zhang, *Theoretical and experimental studies of the flux-flow type Josephson oscillator* (PhD thesis, Chalmers University of Technology 1991) 57.